The Physical Origin of the Fundamental Plane (of Elliptical Galaxies)

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Abstract. I review the basic problems posed by the existence of the Fundamental Plane, and discuss its relations with the Virial Theorem (VT). Various possibilities are presented that can produce the observed uniform departure from *homology* (structural, dynamical) and/or from a constant *stellar* mass-to-light ratio. The role of orbital anisotropy and its relation with the FP thickness are also discussed. None of the explored solutions – albeit formally correct – are easily acceptable from a physical point of view, due to the ever-present problem of the required fine-tuning.

1 Observational Facts

Three are the main global observables of elliptical galaxies (Es): the central projected velocity dispersion σ_0, the effective radius R_e, and the mean effective surface brightness within R_e, $\langle I\rangle_e = L_B/2\pi R_e^2$. It is well known that Es do not populate uniformly this three dimensional parameter space; they are rather confined to a narrow logarithmic plane (Dressler et al. 1987; Djorgovski and Davis 1987), thus called the *Fundamental Plane* (FP). For example, for Virgo ellipticals

$$R_e \propto \sigma_0^{1.4}\,\langle I\rangle_e^{-0.85}\ . \qquad (1)$$

Bender, Burstein, and Faber (1992, hereafter BBF) have introduced the k coordinate system, in which the new variables are a linear combination of the observables:

$$k_1 \equiv (2\log\sigma_0 + \log R_e)/\sqrt{2}\ , \quad k_2 \equiv (2\log\sigma_0 + 2\log\langle I\rangle_e - \log R_e)/\sqrt{6}\ , \qquad (2)$$

$$k_3 \equiv (2\log\sigma_0 - \log\langle I\rangle_e - \log R_e)/\sqrt{3}\ . \qquad (3)$$

In the new space, the k_1-k_3 plane provides an almost edge-on view of the FP, and $k_3 = 0.15k_1 + \text{const.}$ (Fig. 1 of BBF). Here I consider some of the implications of the two main properties of the FP of Virgo and Coma cluster ellipticals: 1) the FP is remarkably thin, with a 1-σ dispersion $\sigma(k_3) \simeq \pm 0.05$ (Bender, private communication); and 2) this thickness is nearly constant along the FP. First, I will try to clarify some frequently misunderstood points about the relations between the FP and the VT, and the meaning of structural/dynamical non-homology.

2 The FP and its relation with the VT

The characteristic dynamical time of Es (e.g., within R_e) is

$$T_{dyn} \simeq (G < \rho >_e)^{-1/2} \approx 10^8 \mathrm{yrs} \ , \tag{4}$$

and their collisionless relaxation time is of the same order (Lynden-Bell 1967), i.e., both are short with respect to the age of Es. As a consequence, only highly perturbed galaxies are presumably caught in a non-stationary phase. Stationarity is a *sufficient* condition for the validity of the VT, and so for Es the Virial Theorem holds. For a galaxy of total stellar mass M_* embedded in a dark matter halo of total mass M_h, the scalar VT can be written as

$$< V_*^2 >= (G\Upsilon_* L_B/R_e) \times (|\tilde{U}_{**}| + \mathcal{R}|\tilde{W}_{*h}|) \ , \tag{5}$$

where $\mathcal{R} \equiv M_h/M_*$ and $\Upsilon_* = M_*/L_B$ is the *stellar* mass-to-light ratio, here defined using the galaxy total *blue* luminosity. The dimensionless functions $\tilde{U}_{**}$ and $\tilde{W}_{*h}$ are the stellar gravitational self-energy and the interaction energy between the stars and the dark matter halo. They depend only on the stellar density profile and on the relative distribution of the stellar matter with respect to the gradient of the dark matter potential, i.e., the dimensionless function on the r.h.s. of equation (5) *depends only on the galaxy structure* (see, e.g., Ciotti, Lanzoni, and Renzini 1996, CLR). Moreover, $\tilde{U}_{**}$ and $\tilde{W}_{*h}$ are known to be *weakly dependent* on the particular density profiles (see, e.g., Spitzer 1969, Ciotti 1991, Dehnen 1993). Obviously the same comments apply also to the stellar velocity virial velocity dispersion $< V_*^2 >$, that necessarily results to be *independent* of the particular internal dynamics of the galaxy (e.g., the amount of orbital anisotropy).

$< V_*^2 >$ is related to σ_0^2 through a dimensionless function that depends on the galaxy structure, its specific internal dynamics and on projection effects:

$$< V_*^2 >= C_K(structure, anisotropy, projection) \times \sigma_0^2 \ . \tag{6}$$

It is important to note that – also in absence of orbital anisotropy – C_K *is very sensitive to galaxy-to-galaxy structural differences*, much more than $\tilde{U}_{**}$ and $\tilde{W}_{*h}$, because it relates a weakly structure dependent quantity ($< V_*^2 >$) to a *local* one (σ_0^2). So, in practice structural non-homology always implies a *strong* variation in the σ^2 profile. Some authors call this phenomenon *dynamical* non-homology, but I (strongly!) suggest to call this effect *kinematical* non-homology, at least for two reasons. First, it is not very useful to call with two different terms (structural vs. dynamical non-homology) the *same* phenomenon: you cannot break structural homology without breaking also kinematical homology. Second, *dynamical* homology should be used to describe galaxies (for example) with the same relative amount of ordered rotation or orbital anisotropy. So, according to this nomenclature, kinematical non-homology can be induced by structural non-homology and/or dynamical non-homology. Moreover, globally isotropic galaxies with different density profiles are structurally non-homologous but dynamically homologous, their $R_e < V_*^2 > /G\Upsilon_* L_B$ are similar, but their C_K's can be significantly

different. As a final remark, it is important to note that the strong dependence of C_K on galaxy structure is essentially due to its definition: in fact, using larger and larger aperture velocity dispersions instead of the *central* velocity dispersion σ_0^2, the projected Virial Theorem is better and better approximated, and for a spherical system without dark matter $C_K \to 3$ independently of the galaxy orbital structure (e.g., Ciotti 1994; cfr. also the results of the numerical simulations of mergers of Capelato et al. 1995).

From equations (2)-(3) and (5)-(6), defining

$$\Theta \equiv [2\pi G\,(|\tilde{U}_{**}| + \mathcal{R}|\tilde{W}_{*h}|)]/C_K \ , \tag{7}$$

one finally obtains

$$k_1 = \log(\Theta \times \Upsilon_* \times L_B/2\pi)/\sqrt{2} \ ; \quad k_3 = \log(\Theta \times \Upsilon_*)/\sqrt{3} \ . \tag{8}$$

Note that the VT does not imply any FP, in fact for fixed L_B different galaxies, all satisfying the VT, can in principle have very different Θ and Υ_*, and so be scattered everywhere in the k-space. So, the statement that *the FP deviates from the VT* – often stated because of the difference between the exponents in eq. (1) and those in the VT [eq. (5)] – is wrong: the FP deviates from *homology* (in a broad sense).

Summarizing, three ingredients are necessary for a class of hot dynamical systems to flatten about a FP: 1) to be virialized, 2) to have similar structures and internal dynamics, 3) to exhibit a small dispersion of Υ_* for any given L_B. Observations tell us that $\Theta \times \Upsilon_*$ is a very well defined function of the galaxy parameters with an intrinsic nearly constant scatter less than 12%. In particular,

$$\delta(\Theta \times \Upsilon_*)/(\Theta \times \Upsilon_*) < 0.12 \ . \tag{9}$$

As galaxies in the BBF sample span a factor ~ 200 in L_B, the tilt corresponds to a factor ~ 3 increase of $\Theta \times \Upsilon_*$ along the FP, from faint to bright galaxies. If Υ_* and Θ are not finely anticorrelated, this implies a very small dispersion, *separately* for both quantities, at any location on the FP. This sets a very severe restriction on $\Theta \times \Upsilon_*$, which translates into strong constraints on the range that each parameter can span at any location on the FP. It is evident that fine tuning is required to produce the tilt, and yet preserve the tightness of the FP. Note also how, from eqs. (8), galaxies with fixed L_B and various internal dynamics, structure, Υ_*, *move along straight lines* in the k-space, with $k_3 = \sqrt{2/3}k_1 + \mathrm{const.}$. The inclination of this line with respect to the FP given by BBF is equal to $\arctan(\sqrt{2/3}) - \arctan(0.15) \simeq 30\,\mathrm{deg}$: this is the reason why in numerical simulations the end-products of the merging of systems initially placed on the FP are found near the FP itself[1]. In conclusion, what is important is not the attempt to understand, perhaps by the finding of a "good" set of observational quantities, why the FP is "distant" from the VT, but, on the contrary, why galaxies are so similar in structure and dynamics, with such a small scatter.

[1] Moreover, this relation between k_3 and k_1 helps reduce slightly the problem of the FP thickness.

3 Exploring Various Possibilities

For simplicity the origin of the FP tilt can be sought in two *orthogonal* directions: either due to a *stellar population* effect, in which case $\Upsilon_* \propto L_B^{0.2}$ and Θ=const, or due to *structural/dynamical* effects, i.e., $\Theta \propto L_B^{0.2}$ and Υ_*=const.

3.1 A stellar origin: changing the IMF

A systematic change of the stellar initial mass function (IMF) is explored in Renzini and Ciotti (1993, hereafter RC). Υ_* is obtained by convolving the present mass of the stars M with the IMF, where $M = M_i$ for the initial mass $M_i < M_{TO}$ (the turnoff mass), and $M = M_R$ (the remnant mass) for $M_i \geq M_{TO}$. For the IMF we adopt $\psi(M_i) = AL_B M_i^{-(1+x)}$, where L_B is the present day blue luminosity of the population, or a multi-slope Scalo IMF with a variable slope for $M < 0.3 M_\odot$. For details see RC.

Changing M_{inf}. In this case we assume a *decrease* of M_{inf}, the lowest stellar mass, for increasing L_B. We found that – in the case of a single-slope IMF – for no value of x small values of Υ_* (characteristic of the FP faint-end) are obtained, unless M_{inf} is unrealistically high. Reducing the slope does not help: for x below ~ 0.65 Υ_* increases again, since then the mass in remnants increases more than how much the mass in the lower main sequence stars is reduced. Only with the multi-slope Scalo IMF a low Υ_* can be realized (Fig. 1 in RC).

Changing the IMF slope. The previous requirement of a low Υ_* at the FP faint-end forces the choice of a low x, but then Υ_* is quite insensitive to variations of M_{inf}; only for a steep IMF Υ_* is sensitive to M_{inf}. As a consequence a mere variation of M_{inf} with a constant IMF slope cannot account for the observed trend. Hence, a variation of slope is required, by an amount Δx which depends on the adopted M_{inf}. We conclude that *a major change of the IMF slope in the lower main sequence is necessary to account for the FP tilt.* There remains to consider the thickness of the FP. In RC it is shown that in order to preserve the $\sim 12\%$ upper limit on $\sigma(\Upsilon_*)$, the galaxy-to-galaxy dispersion in M_{inf} and x should be extremely small, $< \pm 10\%$ and $< \pm 0.15$, respectively. Such very small galaxy-to-galaxy dispersion, coupled to a large systematic variation of x, is a rather demanding constraint, and we conclude that fine tuning is required to produce the observed tilt of the FP, while preserving its constant thickness: the IMF should be virtually universal for a given galaxy mass, and yet exhibit a large trend with galaxy mass. A more accurate analysis of this scenario, using stellar population synthesis models, is given by Maraston (1996).

3.2 A structural/dynamical origin

In this case, assuming Υ_* =const., we explore under which conditions structural/dynamical effects may cause the tilt in k_3 via a systematic increase of Θ

[RC; CLR; Ciotti and Lanzoni 1996, (CL); Lanzoni and Ciotti 1996 (LC)]. In all these investigations spherical, non rotating, two–component galaxy models are constructed, where the light profiles resemble the $R^{1/4}$ law when projected. The internal dynamics is varied using the Osipkov-Merritt formula. Here for shortness reasons only the main results are summarized.

Dark matter content and distribution. In this case we assume *global isotropy*, i.e., all models are *dynamically homologous*, and we ascribe all the FP tilt to systematic variations of $\mathcal{R} = M_h/M_*$ or $\beta = r_h/r_*$ (r_h is a characteristic radius of the dark matter distribution). Obviously, the larger β, the larger the variations of $\mathcal{R}$ that are required to produce the tilt. For Hernquist+Hernquist models and Hernquist+Plummer models, for $\beta \simeq 5$ exceedingly large values of $\mathcal{R}$ are required to produce the FP tilt ($\mathcal{R} \simeq 30 - 175$). An increasing $\mathcal{R}$ may be at the origin of the observed tilt, provided that $\beta < 5$. The same problem affects all the Jaffe+Quasi-isothermal models that we have considered, for every values of β and r_t. For Jaffe+Jaffe models, $\mathcal{R}$ never becomes larger than 10 thus every value of this parameter is acceptable, for every explored value of β.

Structural non-homology. Systematic deviations of the Es light distribution from the standard $R^{1/4}$ profile may also possibly cause the FP tilt (Djorgovski 1995; Hjorth and Madsen 1995). We investigate this *morphological* option using isotropic $R^{1/m}$ models without dark matter, thus ascribing to a systematic variation of m the origin of the tilt. Assuming the faintest galaxies to be $R^{1/4}$ systems, in order to produce the tilt m has to increase from 4 up to ~ 10 along k_1 (CLR). If one assumes $m = 2$ for the faintest galaxies, the required variation is even larger, about a factor of 4, up to ~ 8, and its permitted variation at each FP location remains very small: a scatter of $m < 10\%$ at any location on the FP should be associated to a large variation of it with galaxy luminosities. A systematic trend of m with galaxy luminosity has been reported (Caon, Capaccioli and D'Onofrio 1993), with m increasing from ~ 1 up to ~ 15, thus spanning a much wider range than required to produce the tilt. We conclude that further observational studies are required in order to determine whether a progression of light-profile shapes along the FP really exists among cluster ellipticals.

The role of anisotropy. In this case we ascribed the entire tilt of the FP to a trend with L_B in the anisotropy degree of the galaxies (described in the Osipkov-Merritt formulation by the anisotropy radius r_a), assuming no dark matter. For Hernquist, Jaffe, and $R^{1/m}$ models constrained to the FP we found that, above a certain luminosity, the phase–space distribution function runs into negative values. So we conclude that anisotropy alone cannot be at the origin of the tilt, because the extreme values of r_a that would be required correspond to dynamically inconsistent models (CLR, CL, LC). A special problem with anisotropy is raised by the FP thinness: for galaxies with a positive distribution function a strong fine tuning between L_B and r_a could appear to be required. However

in CL and LC we showed, using a semi–quantitative global instability indicator, that in $R^{1/m}$ models the excursion in anisotropy permitted by *stability* is much less than that given by the simple dynamical consistency, and the induced scatter on the FP is inside the observed spread in k_3. We are well aware that this result is very qualitative (where is it placed in the k–space the end-product of a radially unstable galaxy model initially on the FP?), and so we feel that this result is worth to be further studied, using N-body simulations.

4 Conclusions

Our exploration indicates that all structural/dynamical/stellar population solutions to the FP tilt are rather unappealing, though some are more so than others. This comes from the strong *fine tuning* that is required, no matter whether the driving parameter is the amount of dark matter ($\mathcal{R}$), its distribution relative to the bright matter (β), the shape of the surface brightness distribution (m), or finally the properties of the IMF. In addition to this, we have excluded a trend in the anisotropy as possible cause of the tilt, because it leads to physically inconsistent models. Finally, we showed in a semi-quantitative way that probably a fine tuning of anisotropy with the galaxy luminosity is not required for stability arguments, but deeper investigations, both analytical and numerical, are required.

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The Fundamental Plane of Galaxy Clusters

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Abstract. In the three–dimensional space defined by the logarithms of central velocity dispersion σ, effective radius R_e and mean effective surface brightness I_e, elliptical galaxies are confined in a narrow plane (Dressler et al. 1987; Djorgovski & Davis 1987). Here we discuss the observational evidence for the existence of an analogous relation for galaxy clusters (Schaeffer et al., 1993).

The relations between global observables in stellar systems are important both from a theoretical and a practical point of view. The Tully–Fisher relation for spirals and the Faber–Jackson relation for ellipticals involve two observables. In the case of ellipticals, the residual scatter suggested the introduction of a third parameter, resulting in the definition of the so–called fundamental plane (Djorgovski & Davis 1987; Dressler et al. 1987). From the virial theorem we expect a relation involving the three observables R, L and σ, which can be expressed in the following way (see Djorgovski & Santiago 1993): $L \propto KR\sigma^2(M/L)^{-1}$ where K is a structural parameter. Therefore the existence of the FP for a given class of objects is not a trivial consequence of the virial theorem; it requires also that the class of objects under study has a similar dynamical structure and a tight mass to light ratio with a small dispersion.

We decided to search for relations between global observables in galaxy clusters (Schaeffer et al. 1993). We used the effective radii and total luminosities for a sample of 29 *regular* galaxy clusters, measured after an accurate and homogeneous reduction of high–quality photometric data by West et al. (1989; WOD), who had found a well–defined radius–luminosity relation, $R \propto L^{0.5}$. For 16 of these clusters we found reliable measures of velocity dispersion (Struble & Rood 1991). Combining these data we showed the existence of a relation between velocity dispersion and luminosity, $L \propto \sigma^{1.9}$. The relation L–σ for clusters is the equivalent of the Faber–Jackson law for ellipticals, but in clusters the luminous matter in stars is a small fraction of the total mass, dominated by dark matter and the hot gas. We realized also that the relations R–L and L–σ had a scatter larger than one could expect from observational errors. Introducing σ as a third parameter, we found that these clusters define a Fundamental Plane (fig.1), as many stellar systems from globular clusters (Nieto et al. 1990) to elliptical galaxies (see fig.3 in Schaeffer et al. 1993). We found $L \propto R^{0.89\pm0.15}\sigma^{1.28\pm0.10}$ or, defining a surface brightness $I_e = L/R_e^2$, $R_e \propto {I_e}^{-0.81}\sigma^{1.15}$, with the best fit parameters quite similar to the elliptical ones. The cluster FP is obviously shifted relatively to the elliptical one, because of the different M/L ratio. Therefore all gravitationally bound systems appear to define their FP. The common

link is the virial theorem, but of course the slope and dispersion of each plane will depend on the class of objects taken into account. Might the cluster FP be the result of spurious effects? Discussing their R–L relation for galaxy clusters, WOD excluded a bias due to a selection in surface brightness, as one Abell radius includes most of the cluster luminosity. We also note that cluster peculiar velocities cannot produce the observed relation. Furthermore, tests with numerical simulations (Pentericci et al. 1996) show that the cluster FP is conserved after merging and that it cannot be spuriously generated by the procedure used to find R_e and L.

One can also use the coordinates defined by Bender et al. (1992) (fig.1). These coordinate system, which does not correspond exactly to the FP, gives $k_3 \propto \log(M/L)$ vs. $k_1 \propto \log(M)$. It is clear that the M/L of clusters has a small dispersion (see the discussion in Renzini & Ciotti 1993). We find a small trend of M/L with L, with $M/L \propto L^{0.3\pm0.1}$.

We can also have a rough estimate of the cluster peculiar velocity $V_p = V_{obs} - H_0 D$ (assuming that the deviation from the FP is entirely due to V_p, and neglecting the scatter of the M/L ratio). Uncertainties become very large beyond $z = 0.05$; for the 10 clusters at $z \leq 0.05$ we find (Cappi et al. 1994) $V_p \leq 1000$ km/s, with a trend as a function of the cosine angle between clusters and the GA direction consistent with the results of Han & Mould (1992); the upper limit $\Delta H/H \leq 15\%$ is consistent with Lauer & Postman (1992).

We conclude that regular clusters define a FP comparable to that of elliptical galaxies, which can give us information about formation time dispersions, M/L ratios, or cluster v_p. A critical point is the quality of the data, especially the photometry. For example, a number of recent data give measures of virial radii significantly underestimated, because based on a limited region of the cluster (as discussed by Carlberg et al. 1996). One should remind that typical effective radii are around 1 h^{-1} Mpc, while virial radii are more than 2 times R_e. Therefore a large observational effort is still required to address many important issues, and we need CCD photometric surveys covering a large part of each cluster and not only its central regions. Last but not least, the combination with X–ray data will be extremely useful to understand the role of the different matter components in galaxy clusters.

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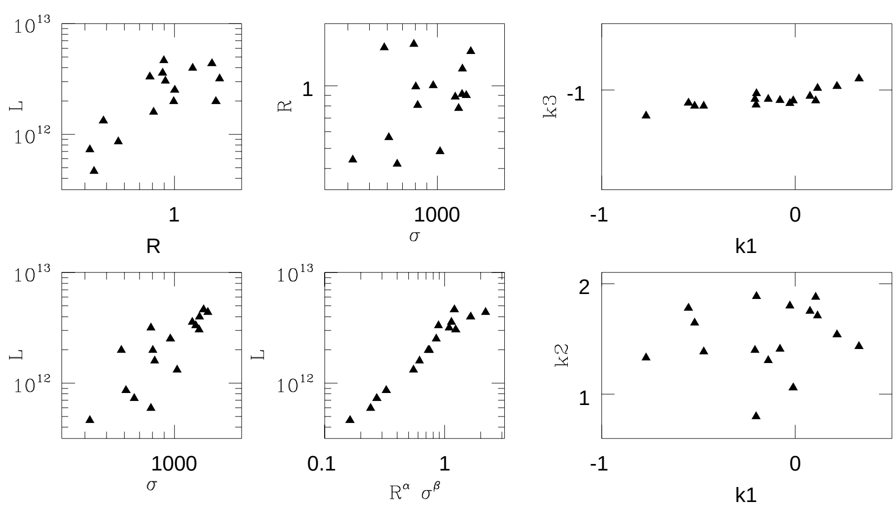

Fig. 1. Relations between L, R and σ, and between k_1, k_2 and k_3

Anisotropic $R^{1/m}$ Models. Velocity Profiles, and the FP of Elliptical Galaxies

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Abstract. We study the dynamical properties of spherical galaxies with surface luminosity profile described by the $R^{1/m}$-law, in which a variable degree of orbital anisotropy is allowed. The stability of the models against radial–orbit instability is studied, and the limits on the maximum anisotropy allowed for each model are determined. The consequent constraints on the the projected velocity dispersion imply that no fine-tuning for anisotropy is required along the fundamental plane (FP) in order to maintain its small thickness. Finally, the velocity profiles are constructed, and their deviations from a gaussian discussed.

1 The models

The models surface brightness profile is described by the $R^{1/m}$ law (Sersic 1968):

$$I(R) = I_\circ \exp\left[-b(m)\,(R/R_{\rm e})^{1/m}\right], \tag{1}$$

which seems to give a better fit to the spheroidal galaxies surface brightness profiles (e.g., Caon et al. 1993; Graham et al. 1996, and references therein) than the "standard" $R^{1/4}$ law [de Vaucouleurs 1948; Eq.(1) for $m = 4$].

The velocity dispersion tensor of our models is described by the Osipkov–Merritt parameterization (Osipkov 1979; Merritt 1985), and characterized by the *anisotropy radius* $r_{\rm a}$: in the limit $r_{\rm a} \to \infty$ the velocity dispersion tensor is globally isotropic, while in general the radial anisotropy increases with radius.

2 Stability

The stability of the anisotropic models is investigated in a semi-quantitative way using the radial-orbit instability indicator ξ (e.g., Fridman and Polyachenko 1984). A model is likely to be unstable if

$$\xi \equiv \frac{2K_{\rm r}}{K_{\rm t}} \gtrsim 1.5 \div 2, \tag{2}$$

where $K_{\rm r}$ and $K_{\rm t}$ are the radial and the tangential kinetic energies, respectively. This parameter is quite independent of the assumed density distribution profile, and for any globally isotropic system $\xi = 1$, while in presence of radial anisotropy $2K_{\rm r} > K_{\rm t}$, and so $\xi > 1$.

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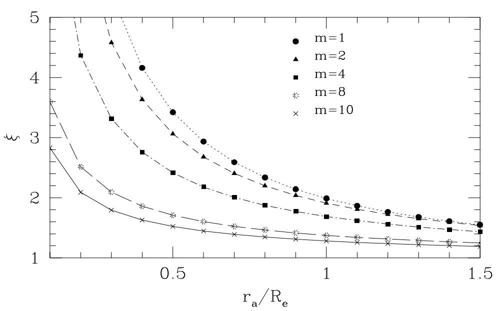
For the investigated models, $\xi = \xi(r_a, m)$, and for a fixed m it decreases towards unity for increasing r_a (see **Fig.1**), according to the previous discussion. So, assuming a fiducial critical value of ξ for stability (e.g., $\xi = 1.7$), a minimum value for the anisotropy radius $(r_a)_\xi$ is obtained, i.e., all models with $r_a < (r_a)_\xi$ are unstable.



For stable models we compute the spatial velocity dispersion profile by integration of the Jeans equation. After the operations of projection and mean, we obtain the *aperture* velocity dispersion σ_a, which mimics the observed central one (see Ciotti and Lanzoni 1997).

3 Implications for the FP

In principle, one of the possible origins of the FP tilt could be a systematic increase of radial anisotropy from faint to bright galaxies (Ciotti et al. 1996). If this is the case, a variation of factor 3 from isotropic to anisotropic squared central velocity dispersion σ_a^2 is required, in the assumption of structural homology (i.e., the same m for all galaxies). On the contrary, for our models we find that, if the maximum degree of anisotropy allowed by the stability requirement is considered, the *radial anisotropy cannot produce the tilt* (a conclusion already reached for different galaxy models in Ciotti et al. 1996).

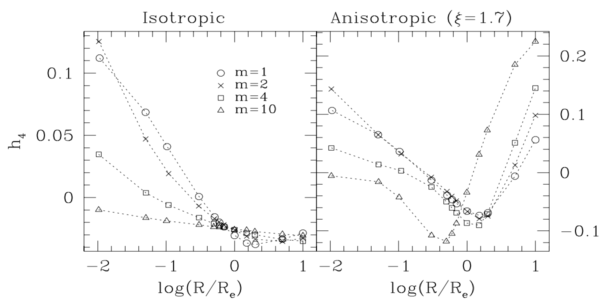
As concerns the problem of the very small thickness of the FP, the limits imposed by the stability requirement imply that the variations between the isotropic and the maximum anisotropic velocity dispersion are so small that the *anisotropy is not required to be fine-tuned with the galaxy luminosity in order to maintain the small observed FP scatter* (see Ciotti and Lanzoni 1997).

4 Velocity Profiles

The velocity profile (VP) at a certain projected distance from the galaxy center is the distribution of the stars line–of–sight velocities at that point. The analysis of the deviations of VPs from gaussianity may give important insights on the dynamical structure of a galaxy (e.g., van der Marel 1994).

We numerically recover the VPs of our models (as in Carollo et al. 1995), and expand them on the Gauss-Hermite basis (Gerhard 1993; van der Marel and Franx 1993), thus obtaining the values of the coefficient h_4 at various distances from the galaxy center. Generally, a negative h_4 indicates a flat–topped VP, while a positive one indicates a VP more centrally peaked than a Gaussian.

In **Fig.2** the radial behaviour of h_4 for various m and for isotropic (left panels) and anisotropic ($\xi = 1.7$, right panels) models is shown.



Because of the differences in the trend of h_4 between the isotropic and the anisotropic cases, and because of the growing evidence that $R^{1/m}$ law appropriately describe the surface brightness profiles of elliptical galaxies, we suggest that a detailed study of the VPs along the FP could be in principle a tool to study the effect of orbital anisotropy on its tilt and thickness.

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Stellar Populations in Galaxies

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Abstract. An innovative tool for the construction of Evolutionary Synthesis models of Stellar Populations is presented. It is based on three independent matrices giving respectively 1) the fuel consumption during each evolutionary phase as a function of stellar mass, 2) the typical temperatures and gravities during such phases, and 3) colors and bolometric corrections as a function of gravity and temperature. The modular structure of the code allows to easily assess the impact, on the synthetic spectral energy distribution, of the various assumptions and model ingredients. As an illustrative example, a solar model (Y=0.27,Z=0.02), with an age ranging between 30 Myr and 15 Gyr is presented and synthetic broad band colours are compared with Magellanic Clouds Globular Clusters data. Then, the evolution of the stellar mass-to-light ratios has been computed and the implications with respect to the properties of the Fundamental Plane (FP) of elliptical galaxies are briefly discussed.

1 Introduction

In the last years, many authors devoted themselves to the Evolutionary Synthesis of Stellar Populations and two main approaches have been followed. The first, known as "Isochrone Synthesis" technique (e.g.Bruzual & Charlot, 1996), simply consists in summing up the monochromatic luminosities contributions of all the mass-points along an isochrone, having assumed an Initial Mass Function. The integration terminates to the latest point on the isochrone itself, that usually coincides with the end of the so-called Early Asymptotic Giant Branch phase: later stellar phases are then added, in an analitycal or empirical way, with some sort of individual receipts. An alternative approach is that introduced by Renzini & Buzzoni (1986) and based on the so-called *Fuel Consumption Theorem*. In this case, the main ingredient for the synthesis is the nuclear fuel available for a star in a peculiar evolutionary stage. This quantity, obtained from the stellar tracks, is then converted in the various luminosities, using a temperatures/gravities-colours set of tranformations. At present, this method has been followed by Buzzoni (1989) and Worthey (1992), but they both offered models only in a narrow age range, missing young and intermediate age populations. In this work we have computed simple population models in the age range 30 Myr-15 Gyr and compared them with the globular cluster family of the Magellanic Clouds; then, the evolution of the global stellar mass-to-light ratios and its consequences with regards to the tilt/thickness of the Fundamental Plane of cluster elliptical galaxies are discussed. For a complete description of the technique see Maraston & Renzini (1997).

2 A test for Simple Stellar Populations models

The synthetic broad band colors have been compared with the data relative to the Magellanic Clouds Globular Clusters and with the results of some other authors (Bruzual & Charlot, 1996 and Tantalo et al. 1996). We have chosen two-colours diagrams instead of age-colours ones just because these don't make use of any age-relations, that are extremely stellar tracks dependent and so a good agreement can be achieved simply varying the set in using. While, in the case of UBV indexes, the three sets of models evolve in much the same way in time and show a quite good agreement with the observations, (Fig. 1a), the infrared colours behave very differently. In particular, the other models fail in reproducing the observed locus for the Magellanic Clouds clusters in the (V-K) *vs.* (U-B) diagram (Fig. 1b). This is due to our correct inclusion of the *TP-AGB* phase, because this stellar stage has a strong impact on the integrated $(V-K)_0$ colour at intermediate age. The disagreement between models and observational data at $(B-V) > 0.5$ in panel (a) or at the later SWB types in panel (b) is due to the increasing difference in metallicity between the older clusters and the solar model.

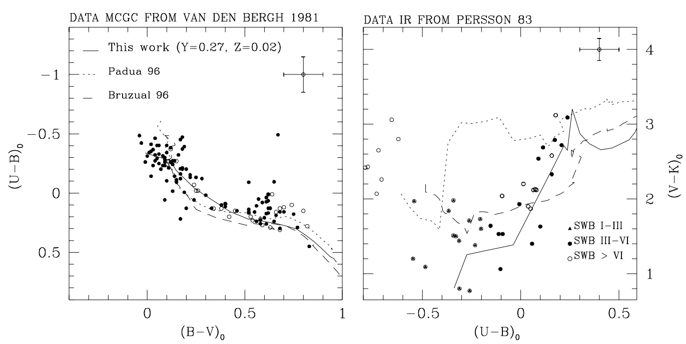

Fig. 1.

3 The rôle of the IMF on the FP properties

To couple the topic of this conference, we have re-analysed the discussion of Renzini & Ciotti (1993, RC93), concerning the rôle of the IMF slope (x), of the lower Main Sequence cut-off (M_{inf}) and of the luminosity time evolution, on the apparent trend of M_*/L_B for cluster ellipticals (CE). In our computations, following RC93, mass of "living" stars plus mass in remnants has been considered. Fig. 2 shows M_*/L_B, for a 15 Gyr population, as a function of M_{inf} for several choices of a Scalo multislope IMF, with x being the esponent for the low MS ($M < 0.3M_\odot$) component. The *observable narrowness* of the FP at low luminosities (which results in M_*/L_B spread less than $\simeq 12\%$), is easily reproducible

as low M_*/L_B values are quite insensitive to variations both in M_{inf} and x. For what it concerns the FP *tilt*, which implies an increase of a factor of $\simeq 3$ in M_*/L_B when passing from faintest to brigthest CE's, a simple variation of M_{inf}, with fixed x, doesn't suffice, but a major change in slope is required. So, to simultaneously obtain a very small galaxy-galaxy dispersion (at the same luminosity/mass) and the tilt, seems to require a low-MS IMF slope that changes very much with L_B, but again fine-tuned with M_{inf}. Finally M_*/L_B evolve in time, because so do luminosities: Fig. 2 (b) shows that the FP thickness impose a very narrow range in galaxy ages, with a dispersion of $\simeq 1.3$ Gyr (for $t > 10 Gyr$). So we confirm, in a more quantitative way, the preliminary results of RC93.

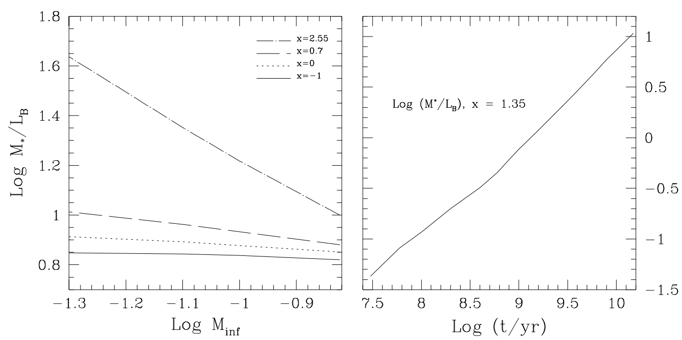

Fig. 2.

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The Relation between X-ray Emission, Galaxy Structure and Internal Kinematics in Early-type Galaxies

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Abstract. Using data in the literature plus new spectroscopic observations, we have built the first large sample of X-ray emitting early-type galaxies with known kinematics (central velocity dispersion $\sigma_{\rm c}$ and maximum rotational velocity $v_{\rm rot}$). With this sample we investigate the effect of rotation on the X-ray emission, particularly with regard to the X-ray underluminosity of flat systems. The role of structural parameters as the ellipticity ϵ, the isophotal shape parameter a_4/a, and the inner slope of the surface brightness profile γ recently measured by *HST* are also investigated.

1 The Problem

The large scatter in the $L_{\rm X}-L_{\rm B}$ diagram is the most striking feature of the X-ray properties of early-type galaxies: it well reaches two orders of magnitude in $L_{\rm X}$ at any fixed $L_{\rm B} > 3\ 10^{10} L_\odot$ (Fabbiano et al. 1992). It was originally explained by environmental differences (White & Sarazin 1991), or by different dynamical phases for the hot gas flows, ranging from winds to subsonic outflows to inflows (Ciotti et al. 1991). Recent observational results have produced a new debate on the explanation of the scatter. Eskridge et al. (1995a,b) showed that on average S0s have lower $L_{\rm X}$ and $L_{\rm X}/L_{\rm B}$ at any fixed $L_{\rm B}$ than do Es. Moreover, galaxies with axial ratio close to unity span the full range of $L_{\rm X}$, while flat systems all have $L_{\rm X} \lesssim 10^{41}$ erg s^{-1}; this holds both for Es and S0s separately. Has the shape of the mass distribution any role in determining $L_{\rm X}$? Since flat galaxies exhibit a higher rotation level, what is the effect of galactic rotation?

Finally, recent *HST* results show that hot galaxies can be divided into two types: *core* galaxies, described by a surface brightness profile $I \propto r^{-\gamma}$ ($\gamma \le 0.3$) at the center, and *power-law* galaxies, with steep, featureless profiles (Faber et al. 1996). These central properties correlate with global ones as rotation and isophotal shape. Is there any correlation also with global $L_{\rm X}$?

2 Our observations and sample

We obtained measurements of velocity dispersion σ and radial velocity v along the major axis for 7 Es and S0s belonging to the *Einstein* sample (Fabbiano et al. 1992), the largest homogeneous sample of early-type galaxies with measured X-ray emission. The main characteristics of these galaxies, the details of

the spectroscopic observations, the final radial velocity and velocity dispersion profiles, and the kinematic properties of the final sample obtained by collecting data also from the literature are given in Pellegrini et al. (1996, hereafter PHC).

3 Results

3.1 X-ray emission, rotation and flattening

The trend between the X-ray to optical ratio $L_{\rm X}/L_{\rm B}$, a measure of the hot gas content of the galaxies, and $v_{\rm rot}/\sigma_{\rm c}$ is L-shaped, with the X-ray brightest objects confined at $v_{\rm rot}/\sigma_{\rm c} \lesssim 0.4$, both for Es and S0s (Fig. 1, with $L_{\rm X}$ and $L_{\rm B}$ derived as in PHC). The trend between $L_{\rm X}/L_{\rm B}$ and the ellipticity ϵ is also L-shaped, even though less sharp: there are no high $L_{\rm X}/L_{\rm B}$ objects with high ϵ. Instead, there is no clear trend between $L_{\rm X}/L_{\rm B}$ and the anisotropy parameter $(v/\sigma)^*$. So, the observations suggest that the major effect on $L_{\rm X}/L_{\rm B}$ is given by the relative importance of rotation and random motions (see also PHC).

3.2 X-ray emission, isophotal shape, and central brightness profile

Faber et al. (1996) show that core galaxies tend to be boxy and slowly rotating, whereas power-law galaxies tend to be disky and rapidly rotating. Bender et al. (1989) showed that disky objects are low X-ray emitters, while boxy and irregular objects show a large range of $L_{\rm X}$.

The results of our investigation (Fig. 1) show a striking relationship between $L_{\rm X}$ and γ: core galaxies span the whole range of $L_{\rm X}$, while power-law galaxies are confined within $\log L_{\rm X} \lesssim 41$. The origin of this new threshold effect should be looked for in the connection between central structural properties (a central black hole? different formation histories?) and global $L_{\rm X}$.

4 Conclusions

Rotation and flattening are predicted to have different effects in the pure inflow and in the wind/outflow/inflow scenarios. Rotation plays the major role in the first one (Brighenti & Mathews 1996), but the expected correlation of $L_{\rm X}/L_{\rm B}$, or of the residuals of the $L_{\rm X}-L_{\rm B}$ correlation, with $v_{\rm rot}/\sigma_{\rm c}$ is not present (PHC). Flattening is determinant in the second scenario (Ciotti & Pellegrini 1996; D'Ercole & Ciotti 1996), but it remains to be explained why the L-shape is more pronounced with respect to $v_{\rm rot}/\sigma_{\rm c}$ rather than to ϵ.

The most significant effect found is the L-shape in the $L_{\rm X}-\gamma$ plot: the division of early-type galaxies into two types corresponds to a difference also in their X-ray properties. This represents another link between the central structures of hot galaxies and their global properties, that might also be useful to better understand their formation history and evolution.

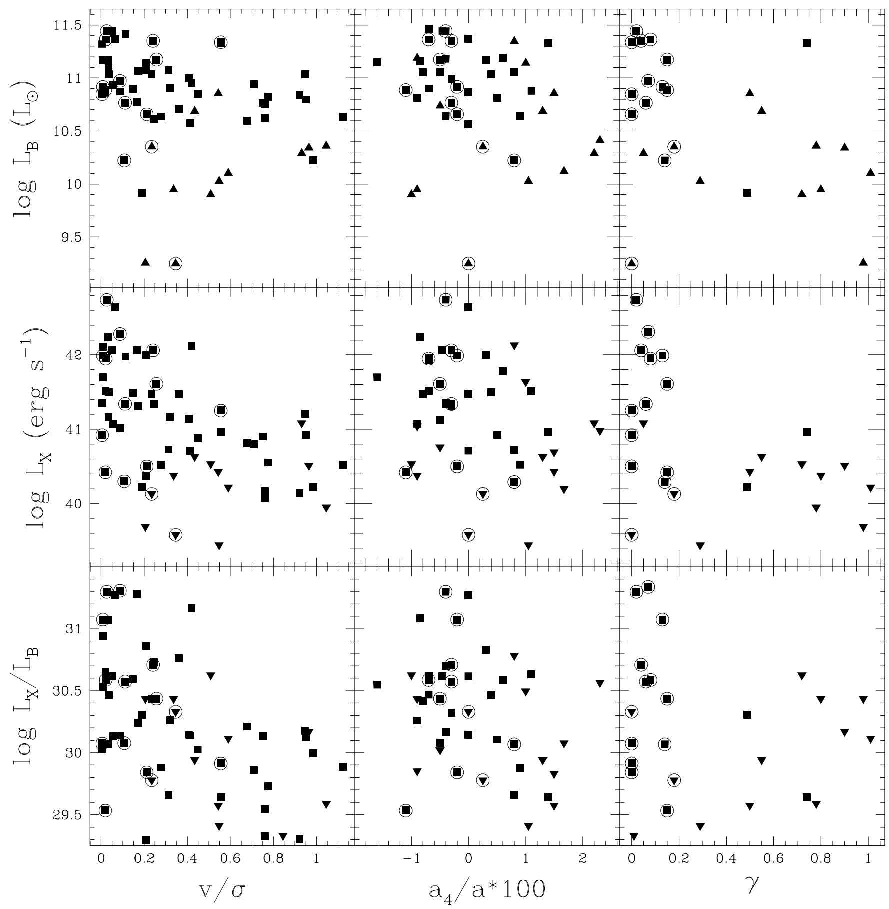

Fig. 1. a_4 is from Eskridge et al. (1995b), γ from Faber et al. (1996). Boxies are X-ray detections, triangles upper limits; *core* galaxies have been circled.

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